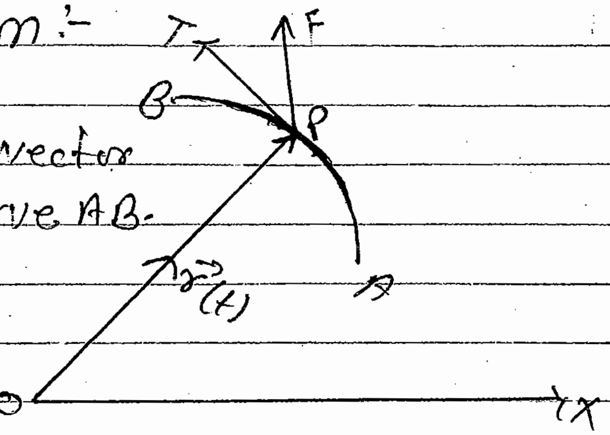
* Integration Of Vectors :-

* Line Integration :-



Let $\vec{F}(x,y,z)$ be a vector function and a curve AB.

"Line integration of a vector function $\vec{F}$ along the curve AB is defined as integral of the component of $\vec{F}$ along the tangent to the curve AB.

Component of $\vec{F}$ along a tangent PT at P

= Dot product of $\vec{F}$ and unit vector along PT

$= \vec{F}\cdot\frac{d\vec{r}}{ds}$ ($\frac{d\vec{r}}{ds}$ is a unit vector along tangent PT)

Line integral $= \sum \vec{F}\cdot\frac{d\vec{r}}{ds}$ from A to B along the curve.

$$\boxed{\text{Line integral} = \int_C \left(F\cdot\frac{d\vec{r}}{ds}\right)ds = \int_C \vec{F}\cdot d\vec{r}}$$

by Sir

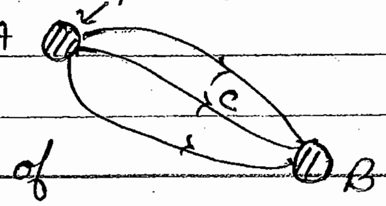
Line Integration :-

Integration of a vector along a curve. C.

Tip $\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$ trace a curve C.

Suppose $\vec{F}(t)$ is vector which define & continuous all the points on the curve C.

$$\boxed{F(t) = \int_A^B \vec{F}\cdot d\vec{r}}$$ ← Mathematical representation of line integration.



It is the workdone by the force on the particle for the displacement from point A to B.

$$\boxed{W_{AB} = \int_A^B \vec{F}\cdot d\vec{r}}$$

If $\vec{\nabla} \times \vec{F} = 0 \Rightarrow \vec{F} = \vec{\nabla}\phi$

If force is conservative -

$$W_{AB} = \int_A^B \vec{F}\cdot d\vec{r} = \int_A^B \vec{\nabla}\phi \cdot d\vec{r}$$

$$= \int_A^B d\phi$$

$\because \phi = \phi(x,y,z)$

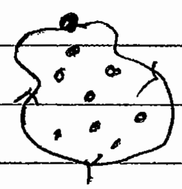
$\Rightarrow d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz.$

$$\boxed{W_{AB} = \phi_B - \phi_A}$$

Workdone by a conservative field is independent of path it only depends on the value of the scalar potential at the initial and final points.

$\Rightarrow$ Workdone by a conservative field along a closed path is zero.

$$\boxed{W = \oint \vec{F}\cdot d\vec{r} = 0}$$



This condition is true only if field $\vec{F}$ is define and continuous at all points on curve and within the curve.